Radial velocity of a sound source in circular motion for illustrating the detection of an exoplanet

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Radial velocity of a sound source in circular motion for illustrating the detection of an exoplanet

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Since the discovery of the first planet outside the solar system in 1995, the detection of exoplanets has been an attractive and engaging scientific field. This article intends to present briefly the radial velocity method for detecting the presence of an unseen planet orbiting a star. Based on an experimental setup, the presentation resorts to the analogy between sound waves and light waves. In particular, they can undergo the Doppler effect from which the radial velocity can be determined. Because the Doppler effect is easier to observe for sound waves than for light waves, a Bluetooth speaker simulates a star. It rotates in a horizontal circle with a constant angular speed, while emitting at two user-selected frequencies, simulating two spectral lines of a star. From the analysis of spectrograms, the radial velocities are deduced. Very good agreement is found between the experiment and theory.
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I. INTRODUCTION

Since its discovery in 1842 and with the advancements of science, the Doppler effect has become a powerful tool to investigate motion with applications in various fields such as medicine,[1] astronomy,[2,3] and radar technology.[4] The Doppler effect occurs for all kinds of waves. It corresponds to the apparent change in the frequency of a wave for a detector moving relative to the source of the wave. When the motion of detector or source is toward the other, the detected frequency is higher than the emitted frequency. However, when the motion of detector or source is away from the other, the detected frequency is lower than the emitted frequency. Whatever the relative motion is, the Doppler effect depends on the component of the velocity that is parallel to the line of sight from the detector to the source, called the radial velocity. Consequently, motion in directions perpendicular to the line of sight produces no Doppler effect.

One application of the physics of the Doppler effect is the detection of extra-solar planets also named exoplanets for short.[2] The direct observation of an exoplanet is very difficult because of the large brightness of the star around which it orbits. Indeed, the reflected light by the exoplanet is drowned out by the glare of the star. For instance, the Sun is 10^9 times brighter than Jupiter, the largest planet in the solar system. To overcome this difficulty, it is possible to indirectly detect a planet by its influence on the star.[2] Its presence can sometimes be inferred from the motion of the star. As the star and the exoplanet orbit their common center of mass, the star moves toward and away from the Earth (see Fig. 1). Its radial motion can be detected by using the Doppler effect for light. This method is most effective when the exoplanet is massive, close to the star, and when its orbital period is short.

The 2019 Nobel Prize in Physics was awarded to three scientists, two of whom, Michel Mayor and Didier Queloz, discovered the first exoplanet called Dimidium.[5,6] 51 Pegasi b is another name for Dimidium.[7] It orbits the star Helvetios, also called 51 Pegasi, similar to the Sun outside the solar system in the constellation Pegasus. 51 Pegasi b is a gas giant similar to Jupiter with nearly half its size. These measurements are based on the Doppler effect for visible light.[2] The basic idea is to compare numerous spectral lines that make up the absorption spectrum of the star with spectral lines of a stationary spectrum in order to measure a shift in the detected frequencies.[8] From the difference between emitted frequencies and detected frequencies, called Doppler shifts, radial velocities of the star are deduced, which provide access to some orbital parameters of the planet. For instance, 51 Pegasi b orbits the center of mass at a distance of ~0.05 AU in a period of 4.2 days whereas Jupiter orbits the Sun at a distance of ~5.2 AU in a period of ~12 years.[2,6,9]

Since the discovery of 51 Pegasi b, the field of exoplanets has been an increasingly popular subject and has an interesting educational value. It gives rise to central issues regarding other worlds in the Universe and the possibility of extraterrestrial life on some other planets.[10,11] These issues may be an opportunity and a motivation for students to learn and apply basic physics concepts in an attractive field. There are useful papers and helpful resources dealing with this theme in order to broaden and deepen our knowledge on the subject.[12–22] However, descriptions of experiments to illustrate the radial velocity method of exoplanet detection have not been widely considered. This aspect is investigated in the present paper by using a sound source that simulates a star. The sound source emitting at two user-selected frequencies moves in a circle at a constant angular speed, with the goal to determine its radial velocity.

II. EXPERIMENT

Figure 2 is a picture of the experimental setup already presented by Saba and Rosa[21] but in a different context. In addition, in the present paper, the measurements are conducted by using a compact Bluetooth speaker controlled by a smartphone application.[23,28] It emits sound waves at 10 kHz and 15 kHz, analogous to a star having different lines in its spectrum. The speaker is firmly fixed at the end of a wood bar, whereas on the opposite end of the bar, a counterweight of similar mass is also firmly fixed. The center of the bar is rigidly attached to the axis of the motor. It is worth noting that all elements must be securely fixed to prevent injury.

The distance from the center of the speaker to the center of the bar is $R = 26.5 \pm 0.5$ cm. The motor controlled by a dc

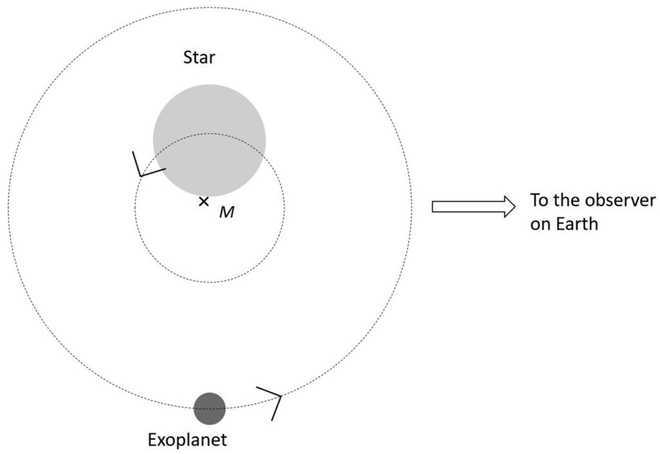

Fig. 1. Top view of the circular motions of a star and an exoplanet about their common center of mass M. The drawing is not to scale. The dashed circles represent the two trajectories. The star and the exoplanet follow orbits having the same shape and period. The distance from 51 Pegasi to the center of mass and the distance from 51 Pegasi b to the center of mass are $\sim 3.2 \times 10^3$ km and $\sim 7.8 \times 10^6$ km, respectively. 51 Pegasi and 51 Pegasi b have a radial velocity amplitude of $\sim 60\,\mathrm{m\ s^{-1}}$ and $\sim 135\,\mathrm{km\ s^{-1}}$, respectively.

power supply spins the bar perpendicular to the vertical axis passing through its center. In this way, the sound source moves with a constant angular speed of $\omega = 42 \pm 1\,\mathrm{rad\ s^{-1}}$ leading to a linear speed of $v = \omega R = 11.2 \pm 0.3\,\mathrm{m\ s^{-1}}$ and a period of revolution of $T = 2\pi/\omega = 0.150 \pm 0.004\,\mathrm{s}$. The linear velocity $\mathbf{v}$ whose magnitude is v is always tangent to the circular path of the point in question (see Fig. 3). The angular speed is monitored by means of a photogate.

A microphone working in the range $30\,\mathrm{Hz} - 20\,\mathrm{kHz}$ is placed in the rotation plane of the source at a distance of $L = 53.0 \pm 0.5\,\mathrm{cm}$ from the center of the bar (see Fig. 3). It simulates the spectrometer on the Earth. It is connected to the input of a sound card on a computer for signal capture in the WAV file format. The analysis of the detected sound is then performed by means of its spectrogram obtained by using free software (WaveSurfer).[24] A smartphone can also be used to record and analyze the sound emitted by the speaker.[25]

III. THEORY

The sound source moves circularly toward and away from the microphone and toward it in a counterclockwise direction

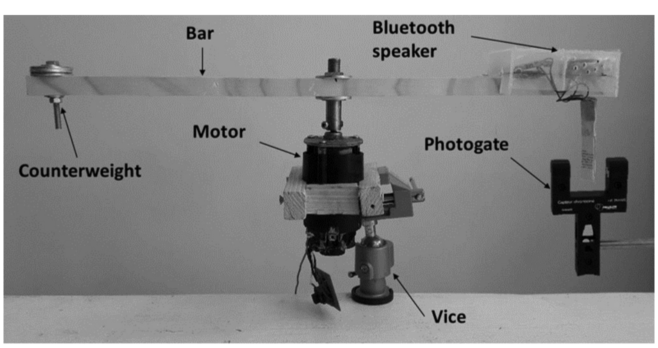

Fig. 2. Photograph of the experimental setup. The speaker is glued at the end of the bar and covered by a plastic box with holes of $\sim$5 mm in diameter. The box is fixed on the bar by using screws. The counterweight consists of a screw, a nut, and some washers.

with a radial velocity $v_r = \mathbf{v} \cdot \hat{u}_r$ (see Fig. 3). Polar coordinates are the most convenient for describing its motion. For these conditions, the detected frequency f and the emitted frequency f_0 are related by

$$f = f_0\left(\frac{v_s}{v_s + \mathbf{v}\cdot\hat{u}_r}\right) = f_0\left(\frac{1}{1+\frac{v_r}{v_s}}\right) \approx f_0\left(1 - \frac{v_r}{v_s}\right), \tag{1}$$

where v_s is the speed of sound in air. Equation (1) indicates that if the source moves away from the microphone ($v_r > 0$), then $f < f_0$, while if the source moves toward the microphone ($v_r < 0$), then $f > f_0$.

By considering the microphone at the origin of the polar coordinate system, the position of the sound source is $\mathbf{r} = r\hat{u}_r$, as shown in Fig. 3.

Given that the velocity vector of the source is defined by $\mathbf{v} = d\mathbf{r}/dt = d(r\hat{u}_r)/dt$, the linear velocity can be written as

$$\mathbf{v} = \dot{r}\hat{u}_r + r\dot{\theta}\hat{u}_\theta. \tag{2}$$

It turns out that the radial velocity is $v_r = \dot{r}$. It should be noted that $\mathbf{v}$ is always tangent to the circle traveled by the sound source and its magnitude is constant.

The law of cosines allows to write the length r as

$$r^2 = L^2 + R^2 - 2LR\cos\omega t, \tag{3}$$

where L, R, and ω are fixed whereas r is a function of time only. The derivative of Eq. (3) with respect to time yields

$$\dot{r} = \frac{L\omega R\sin\omega t}{r}. \tag{4}$$

Finally, substituting for r from Eqs. (3) into (4) and rearranging the result, the radial velocity is given by

$$v_r = \dot{r} = \frac{v\sin\omega t}{\sqrt{1 + m^2 - 2m\cos\omega t}}, \tag{5}$$

where $v = \omega R$ is the linear speed of the sound source and $m = R/L$ is a geometrical factor. The extreme values $v_r = \pm v$ are realized when $\cos\omega t = m$.[26]

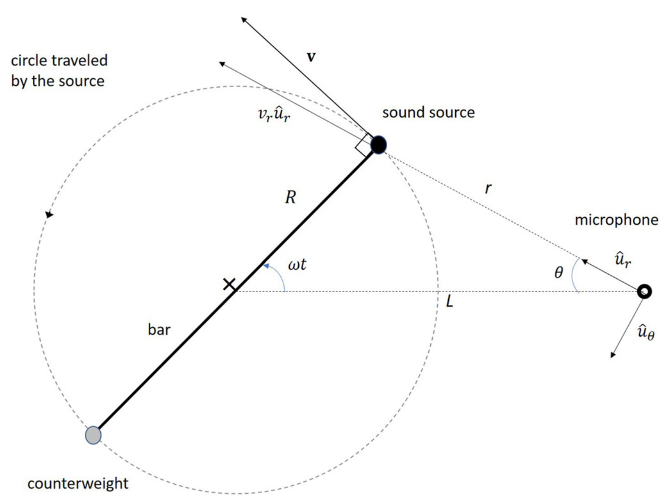

Fig. 3. Schematic representation of the experimental setup viewed from the top. The radial velocity v_r is the radial component corresponding to the projection of the linear velocity $\mathbf{v}$ on the r-axis.

Substituting this expression for v_r into Eq. (1) gives

$$f = f_0\left(1 - \frac{q\sin\omega t}{\sqrt{1+m^2-2m\cos\omega t}}\right), \tag{6}$$

where $q = v/v_s$.

IV. RESULTS AND DISCUSSION

Figure 4 is a screenshot of the software displaying spectrograms of the sound source moving in a circle. The spectrograms show the spectral amplitude, as a function of time (abscissa) and frequency (ordinate), through a gray scale map. The dashed lines represent the two emitted frequencies (a) $f_0 = 15\,\text{kHz}$, and (b) $f_0 = 10\,\text{kHz}$. In both cases, the detected frequencies are periodically and continuously shifted with respect to f_0. In addition, the signal represented in Fig. 4(a) also contains a small-amplitude sinusoidal component around 15 kHz. Given the mathematical form of Eq. (6), one would not expect the signal from the Bluetooth speaker to be a pure sine wave. Consequently, the small amplitude sine wave cannot represent a signal transmitted directly from the speaker to the microphone. It is due to reflection of sound by a surrounding object.[22] That signal can be treated by considering a symmetric and fictitious source located on the other side of the object (method of images).

In order to analyze the variations of the frequency, some data values are extracted from the spectrograms.[27] The extracted data, represented by filled circles and their uncertainties, are plotted with time on the x-axis and frequency on the y-axis, as shown in Fig. 5 (again, the dashed lines indicate the two emitted frequencies). By comparing the Doppler shifts, the amplitude of the frequency oscillation increases as f_0 increases. Indeed, the signal exhibits an oscillation of amplitude 480 Hz for $f_0 = 15\,\text{kHz}$, whereas it shows an oscillation of amplitude 320 Hz for $f_0 = 10\,\text{kHz}$.[28] These observations are consistent with Eq. (1).

The quantitative analysis of these experimental data is carried out by fitting the data with Eq. (6). The solid lines in Fig. 5 represent the best fit curves whose fitting parameters are reported in Table I. The value of the parameters f_0, ω, and m are close to the experimental values indicated in Sec. II, showing good agreement between the theoretical model and the experimental data. From the average value

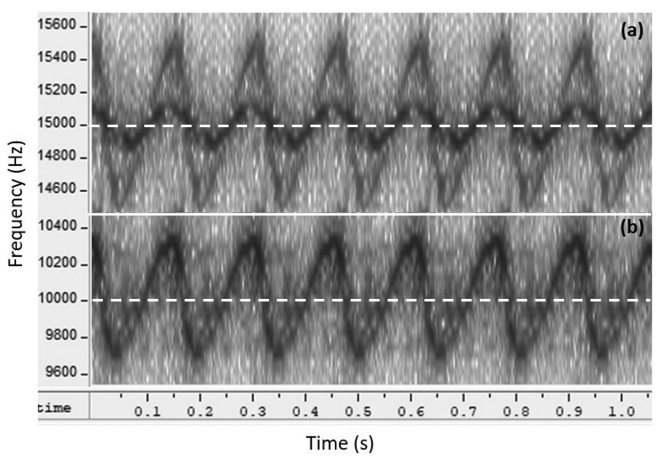

Fig. 4. Spectrograms of the sound source (window length: 1024 points; bandwidth: ~43 Hz) moving in a circle at a constant angular speed and emitting sound (a) at $f_0 = 15\,\text{kHz}$ and (b) at $f_0 = 10\,\text{kHz}$. The dashed lines represent the emitted frequency f_0.

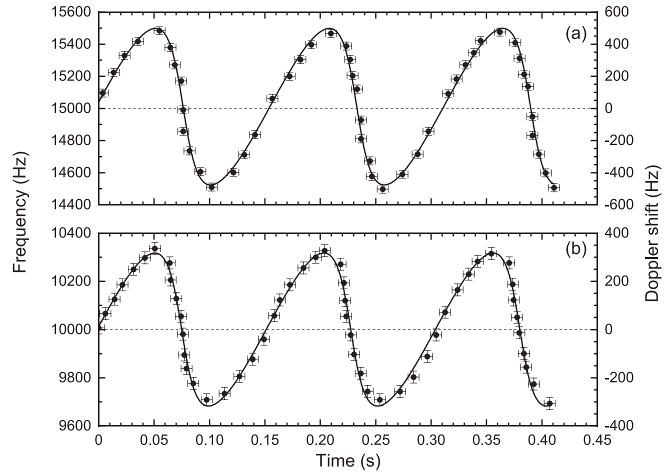

Fig. 5. Data extracted from the spectrograms of Fig. 4. The dashed lines represent the emitted frequency (a) at $f_0 = 15\,\text{kHz}$ and (b) at $f_0 = 10\,\text{kHz}$. The equation of the curves, represented by the solid lines, that best fits all data points within their uncertainties is given by Eq. (6). The error bars are obtained by taking into account the width of the trace obtained with the experimental data of Fig. 4.

$q = 0.0321 \pm 0.0005$ and the linear speed of the sound source $v = 11.2 \pm 0.3\,\text{m s}^{-1}$, the speed of sound is estimated at $v_s = 349 \pm 10\,\text{m s}^{-1}$ in agreement with numerous textbooks as well as handbooks of physical parameters.[29,30]

From the experimental data of Fig. 5 and by using Eq. (1), the radial velocities are calculated. Figure 6 shows the temporal evolution of the radial velocities for the two emitted frequencies studied here. As can be seen, the radial velocities are frequency independent within the uncertainties of the experimental data. The radial velocities vary between the limits $v_r \approx \pm 11\,\text{m s}^{-1}$, as expected. By examining the results of Figs. 5 and 6, the Doppler shift is positive as $v_r < 0$ indicting that the source approaches the microphone. In contrast, the Doppler shift is negative as $v_r > 0$ indicting that the source moves away from the microphone.

In the exoplanet context, most information (period and mass, for instance) is deduced from the evolution of the radial velocity of the star as a function of time. For the purpose of analyzing the temporal evolution of the radial velocity, the data are fitted using Eq. (5). The solid line in Fig. 6 represents the best fit curve over more than two cycles. The result of the fitting procedure is reported in Table II. The obtained parameters are consistent with those reported in the Sec. II.

V. LIMITATIONS AND CONCLUSIONS

This work can be used to show how an exoplanet can be indirectly detected through the motion of its star by using the radial velocity method. A star orbits the center of mass of the star-planet system while emitting light characterized by

Table I. Best-fit parameters obtained by fitting the experimental data of the Doppler shifts of Fig. 5 using Eq. (6). The values are generated by the software.

f_0 (Hz)	ω (rad s^{-1})	$q = v/v_s$	$m = R/L$
$15\,011 \pm 8$	40.1 ± 0.1	0.0324 ± 0.0008	0.54 ± 0.03
$10\,000 \pm 4$	41.2 ± 0.1	0.0317 ± 0.0006	0.54 ± 0.02

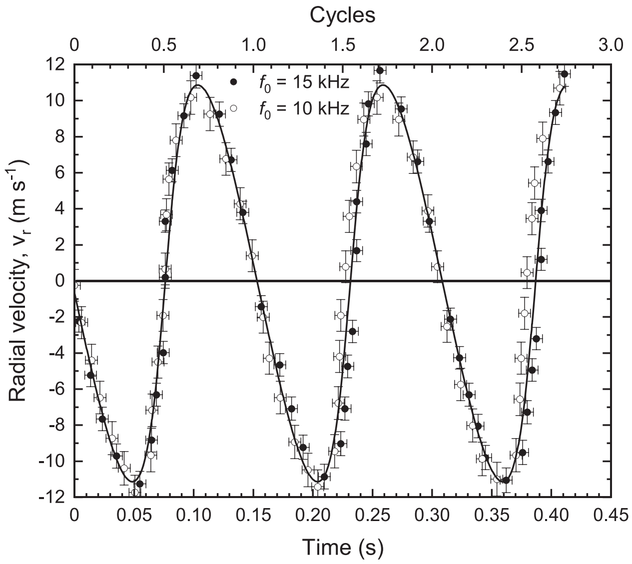

Fig. 6. Temporal evolution of the radial velocity calculated from the data of Fig. 5 by using Eq. (1). The solid line is the best fit to the data using Eq. (5). The uncertainties are computed using Fig. 5, Eq. (1), and standard error propagation.

several spectral lines in the visible range. Similarly, the sound source moves in circle while emitting sound at different frequencies. The common features are the Doppler effect and the circular orbits. The sound source is analogous to the star since both emit waves. Moreover, the microphone that detects the sound from the speaker is analogous to the spectrometer that detects the light from the star.

However, this analogy has some limitations. For instance, the energy emitted from the star is in the form of visible light whereas the energy from the sound source is in the form of sound waves. In addition, while the distance between the spectrometer and the star is many light years, the distance from the microphone to the sound source is a few tens of centimeters. Finally, the orbital period of a star is much larger than the period of the sound source.

In this study, the microphone and the sound source lie in the rotation plane. In practice, many planets orbit with an elliptical path, thus changing the model presented here. It would be interesting to simulate an exoplanet with a different inclination by placing the microphone above or below the speaker orbit plane. This is the strength of the radial velocity method, because exoplanets can be detected when transits are not observable. Moreover, when $L \gg R$, the variation with time of the radial velocity should have a pure sinusoidal shape.

This simple experiment provides students the possibility to model the detection of an exoplanet through its effect on its star's motion. The students obtain experimental data in close agreement with the values predicted from the model. The acoustic Doppler effect experiment described in this paper provides a convenient and familiar analog to the

Table II. Best-fit parameters obtained by fitting the experimental data of the radial velocities of Fig. 6 using Eq. (6). The values are generated by the software.

ω (rad s^{-1})	v (m s^{-1})	$m = R/L$
40.5 ± 0.1	11.0 ± 0.4	0.45 ± 0.05

Doppler effect that alters the frequency of starlight (as observed from the Earth) originating from an actual exoplanet system. It allows students to experimentally measure the Doppler effect of a sound source emitting at different frequencies and moving at a constant speed along a circular path. From the spectrograms of the sound source, they can determine the radial velocities of the source which agree remarkably well with the predictions of the model.

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[8]Astronomical measurements involving light are usually done in wavelengths rather than frequencies. The frequency f is related to the wavelength λ by the expression $\lambda = c/f$, where c is the speed of light. Astronomers use spectrometers with a high spectral resolution $\lambda/\Delta\lambda$ ranging from 50 000 to 100 000 in the visible region.
[9]Jupiter is the most massive and most voluminous of the planets of the solar system. It causes the Sun's velocity to vary with an amplitude of 12.5 m s^{-1}. The precision achieved by the radial velocity method is $\sim$1 – 2 m s^{-1}. Consequently, Jupiter can be detected from another star, like 51 Pegasi. However, it would take 12 years to observe a period.
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[23]Several apps can be found in the Google play store or iTunes store. Here, I used Frequency Sound Generator (Google play).
[24]For instance: Audacity software available at <https://www.audacityteam.org/>, WaveSurfer software available at <https://sourceforge.net/projects/wavesurfer/>, accessed on April 1, 2020.
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[26]As illustrated in Fig. 3, the radial component v_r is the projection of the velocity $\mathbf{v}$ on the r-axis. The magnitude of v_r is maximum when the angle between $\mathbf{v}$ and the r-axis is zero or π. In this case, the sides R and r are perpendicular to each other; they form the legs of a right triangle, with L as hypotenuse. This allows to write $\cos\omega t = R/L = m$. The radial velocity is $v_r = +v$ when $\cos\omega t = m$ and the source moves away from the microphone. Its value is $v_r = -v$ when $\cos\omega t = m$ and the source moves toward the microphone.
[27]The data were extracted with OriginPro software by using the Digitizer tool.
[28]An asymmetry in the frequency shift should appear in the experimental results of Fig. 4. However, the bandwidth of 43 Hz (also the frequency uncertainties) is quite large for detailed investigation. For instance, using $f = f_0(1/1 + (v_r/v_s))$, with $v_r/v_s \approx \pm 0.03$, for $f_0 = 15\,000$ Hz, the minimum frequency is $f_{\min} \approx 14\,563$ Hz corresponding to a shift of -437 Hz with respect to f_0 whereas the maximum frequency is $f_{\min} \approx 15\,464$ Hz corresponding to a shift of 464 Hz. The approximation of Eq. (1) eliminates this asymmetry. Experimentally, a double frequency resolution would make that asymmetry detectable.
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Model Press

This may be some of the apparatus collected in the early years of the University of Mississippi, founded in 1848. In the ante-bellum American South, with its agricultural industry of growing cotton, a press suitable for compressing cotton into bales for shipment would be a useful piece of applied physics. (Picture and text by Thomas B. Greenslade, Jr., Kenyon College)